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EFFECT OF FREE CONVECTIVE HEAT TRANSFER ON THE TEMPERATURE STATE OF SUPERCONDUCTORS

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The article investigates the temperature state of a rigid-type coaxial superconductor when the space of the inner core is filled with stagnant helium.

In designing rigid-type coaxial superconductors the problem arises whether it is expedient to evacuate the space of the inner core. In the present work we investigate the additional axial heat influx to the superconducting cores connected with the free convective flow of helium in the space of the inner core, and the effect of this heat influx on the temperature state of the conductor. This investigation is connected with the tests of the experimental section of the superconductor SPK-100 [1], whose diagram of cryostating is shown in Fig. 1. The current-carrying system SPK-100, consisting of two coaxial cores, is made of copper tubes covered with a layer of superconductor Nb_3Sn. The end sections of the current-carrying system are led into the zone of room temperature, and they form the current lead-ins. The cryoagent helium flows through the coaxial gap, and at the place of contact of the cores with the current lead-ins the flow branches. One part proceeds to cool the current lead-ins, the other returns to the refrigerator. The space of the inner core may be filled with helium at different pressures. The space of the inner core may be filled with helium at different pressures. The heat influx to the cores, which determines the longitudinal temperature distribution, is composed of radial heat influxes through the cryostating shell of the conductor, of heat influxes from the side of the lead-ins due to the thermal conductivity of the metal and the stagnant helium, and also of free convection of the helium. The qualitative temperature distribution along the conductor is shown in Fig. 1. The outer core, far from the current lead-

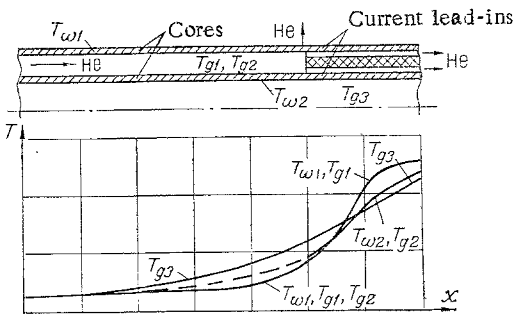

Fig. 1. Diagram of the cryostating of the coaxial conductor and of the temperature distribution: of the outer core T_{W1}, of the inner core T_{W2}, of the stagnant helium T_{g3}, of the cryoagent cooling the CCS T_{g1}, T_{g2} longitudinally.

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in, is acted upon only by radial heat influxes which may be considered small so that the temperature of the outer core T_{w1} is close to the temperature of the stream T_{g1}. Together with the increase of the longitudinal temperature gradient T_{w1}, the transverse heat fluxes also increase in the lead-in of the outer core, but since the heat-transfer coefficients on the lead-in are usually higher than on the core, we may assume that on the outer lead-in $T_{w1} \approx T_{g1}$.

The heat influx through the stagnant helium is expended directly on heating the inner core; therefore, the temperature of the inner core is higher than the temperature of the flowing helium T_{g2}, but lower than the temperature of the stagnant helium T_{g3}. On the inner lead-in the wall temperature T_{w2} is approximately equal to the temperature of the helium cooling it T_{g2}. Over some length of the lead-in, the temperature of the stagnant helium is lower than the temperature of the inner lead-in.

The object of the present work is to devise an engineering method of calculating the temperature of the core at the place of its contact with the lead-in, i.e., the maximum temperature of the superconductor core when the cavity of the inner coaxial tube is filled with helium.

Free convective heat transfer in confined spaces is being widely investigated. However, we do not know of any work whose result could be used for solving the stated problem. We mention only [2, 3], in which a theory is suggested for calculating heat exchange between the end walls in horizontal cavities in laminar gravity flow of a liquid. However, in the case examined by us there are no distinctly fixed end walls with specified temperatures, and the temperature distribution along the core has to be determined from the solution of the equations taking into account the free convection of helium in the space of the inner core, the flow of cryoagent in the annular gap, and the thermal conductivity of the cores themselves.

The engineering method devised by us is based on the following idea. It is assumed that between the flux in the stagnant helium in the horizontal direction and the longitudinal temperature gradient dT/dx there exists the relation

$$q = -\lambda_{\text{equ}} \frac{dT}{dx}, \tag{1}$$

which is similar to Fourier's law for solids. λ_{equ} is the equivalent differential thermal conductivity, which may be determined experimentally and which depends on the core diameter, the temperature gradient, and the thermodynamic parameters of the helium in the given cross section. With the introduction of λ_{equ}, the problem of determining the maximum core temperature is reduced to solving a system of unidimensional differential equations of thermal conductivity for stagnant helium and cores, and energy equations for the cryoagent.

Experimental determination of λ_{equ} and experimental verification of our engineering formula for calculating the maximum core temperature were carried out on a device of the single-phase superconductor 1SPK-M. The value of λ_{equ} obtained was used for calculating the maximum temperature of the experimental section of the SPK-100.

Derivation of the Formula for the Maximum Core Temperature. To describe the temperature distributions along the cores, the following system of equations was used:

$$\begin{gathered}
\frac{d}{dx}\left(\lambda_1 S_1 \frac{dT_{w1}}{dx}\right) - \alpha_1 p_1 (T_{w1} - T_{g1}) + q_1 = 0, \\
G_1 c_{p1} \frac{dT_{g1}}{dx} - \alpha_1 p_1 (T_{w1} - T_{g1}) = 0, \\
\frac{d}{dx}\left(\lambda_2 S_2 \frac{dT_{w2}}{dx}\right) - \alpha_2 p_2 (T_{w2} - T_{g2}) + \alpha_3 p_2 (T_{g3} - T_{w2}) = 0, \\
G_2 c_{p2} \frac{dT_{g2}}{dx} - \alpha_2 p_2 (T_{w2} - T_{g2}) = 0, \\
\frac{d}{dx}\left(\lambda_{\text{equ}} S_3 \frac{dT_{g3}}{dx}\right) - \alpha_3 p_3 (T_{g3} - T_{w2}) = 0.
\end{gathered} \tag{2}$$

Heat exchange between the stagnant helium and the inner tube is regarded as heat exchange between two solids separated by a thin interlayer with finite thermal resistance k, so that $\alpha_3 = 1/k$.

System (2) applies to the case when each tube of the coaxial conductor is cooled by an independent stream of cryoagent. If the cryoagent flows through the coaxial gap, then the energy equation for it is obtained as the sum of the second and fourth equations of system (2), with the condition that $T_{g1} = T_{g2}$.

The equations of system (2) are nonlinear with properties of the material that are very variable along the cores and the lead-ins, and with very variable conditions of heat exchange. To obtain accurate solutions of such equations, numerical methods are used on computers. The method of [4] is known, involving the current carrying system (CCS) of the conductor being divided into n sections; on each section the thermophysical characteristics of the materials and the conditions of heat exchange may be considered to be constant, yet at the boundaries of neighboring sections equality of temperatures and heat fluxes is maintained. If we have the analytical solution for each section obtained for constant properties, and if we take into account the conditions at the boundaries, we can easily obtain the solution of system (2) for variable properties.

However, in the present work we do not use this numerical method of solving system (2); instead, we use only the idea on which it is based. We do not divide the CCS into n sections but only in two: the lead-ins and the cores. For each section the system of equations (2) can be greatly simplified. Then we find the analytical solution of these systems of equations and "join" the solutions, using the natural conditions of equality of temperatures and heat fluxes on the boundary between the cores and the lead-ins. In the process we have to determine the "characteristic" temperature at which the properties of the materials have to be considered. The use of a similar method in investigating cryostatic current lead-ins in [4, 5] lead to satisfactory correspondence of the obtained data with the results of numerical calculation and experiments.

From system (2) we obtain systems of equations for cores and lead-ins, on the assumption that the heat flux densities longitudinally and transversely are sufficiently great in gravity flow of helium in the cavity of the inner core so that we may take it that $k = 0$ and $\lambda_1 s_1 = 0$. With a view to these conditions and with constant properties of the materials, the system of equations (2) for each section is transformed into the form

$$(\lambda_2 s_2 + \lambda_{equ} s_3)\frac{d^2 T_{w2}}{dx^2} - \alpha_2 p_2 (T_{w2} - T_{g2}) = 0,$$

$$G c_p \frac{dT_{g2}}{dx} - \alpha_2 p_2 (T_{w2} - T_{g2}) - q_1 = 0, \tag{3}$$

$$(\lambda_{T2} s_{T2} + \lambda_{Tequ} s_{T3})\frac{d^2 T_{Tw2}}{dx^2} - \alpha_{T2} p_{T2} (T_{Tw2} - T_{Tg2}) = 0,$$

$$G_{T2} c_{pT2} \frac{dT_{Tg2}}{dx} - \alpha_{T2} p_{T2} (T_{Tw2} - T_{Tg2}) = 0. \tag{4}$$

The system of equations (3) is written for the inner core, (4) for the inner lead-in. G is the flow rate of cryoagent to the core; G_{T_1}, G_{T_2} are the flow rates to the lead-ins.

The equations of systems (3), (4) are ordinary differential equations with constant coefficients.

To obtain the formula for the maximum core temperature, i.e., the core temperature $T_w(l)$ at the place of contact of the lead-in with the core, we have to write the solution of system (3) for the temperatures $T_w(l)$ and $T_g(l)$ as a function of the temperatures T_{wo} and T_{go} and of the heat flux from the side of the lead-in $Q_T(l)$, and for the system of equations (4) we have to write the solution for $Q_T(l)$ as a function of the temperatures $T_w(l)$, $T_g(l)$ and of the temperature of the "hot" end of the lead-in in $T_{Tw}(l_T)$. The formula for the maximum temperature follows from these solutions, but it is cumbersome and is not given here. We obtain only a simplified formula for the maximum core temperature of a fairly long, well-cooled conductor, and we verify this formula experimentally.

The temperatures $T_w(l)$ and $T_g(l)$, which follow from the solution of system (3) that is correct when the flow rate is high and/or the conductor is long, have the form

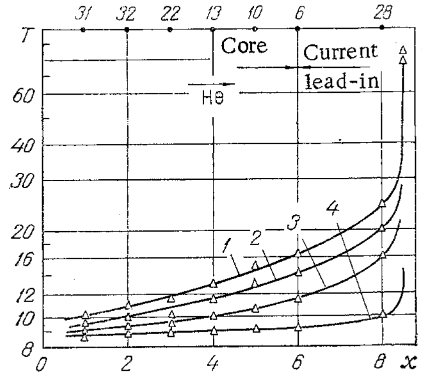

Fig. 2

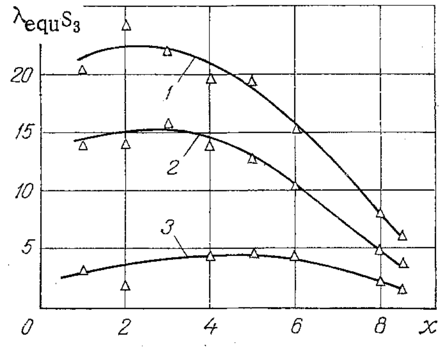

Fig. 3

Fig. 2. Temperature distribution along the inner core of the experimental section of the superconductor 1SPK-M obtained with different pressures of the stagnant helium in the space of the inner core: 1, 2, 3, 4) p = 0.3, 0.2, 0.1, 0 MPa, respectively. On top are the numbers of the temperature sensing elements and the places where they are mounted along the inner core; points indicate experimental data; T, °K; x, m.

Fig. 3. Dependence of the thermal conductivity of stagnant helium $\lambda_{equ}s_3$ (W·m/°K) on the longitudinal coordinate, calculated for curves 1, 2, and 3 in Fig. 2 by formula (10); x, m.

$$T_w(l) = T_{g0} + \frac{fQ_T(l)}{2Gc_p(\sqrt{1+f}-1)}, \tag{5}$$

$$T_g(l) = T_{g0} + \frac{Q_T(l)}{Gc_p}, \tag{6}$$

where $f \equiv (2Gc_p)^2/\alpha p(\lambda s + \lambda_{equ}s_3)$.

The formula for $Q_T(l)$, which is correct when the lead-in is long and the heat-transfer coefficient on the lead-in is high, follows from the system of equations (4):

$$Q_T(l) = \frac{G_T c_{pT}[T_{Tw}(l_T) - T_w(l)]}{1 - \exp g} - G_T c_{pT}[T_w(l) - T_g(l)], \tag{7}$$

where $g = G_T c_{pT} l_T/(\lambda_T S_T + \lambda_{Tequ}s_3)$.

The dimensionless maximum temperature in the section lead-in–core, obtained from (5)-(7), has the form

$$\frac{T_w(l) - T_{w0}}{T_{Tw}(l_T) - T_{w0}} = \left\{1 + \frac{\exp g - 1}{f}\left[f + 2(\sqrt{f+1} - 1)\left(\frac{Gc_p}{G_T c_{pT}} - 1\right)\right]\right\}^{-1}. \tag{8}$$

The subscript 2 was discarded in relations (5)-(8).

Formula (8) makes it possible to calculate the maximum core temperature with known temperatures at the inlet to the cable, at the "hot" end of the lead-in, the known flow rates of cryoagent to the core and lead-in, and the equivalent thermal conductivity of the stagnant helium.

Experimental Determination of λ_{equ} on the Device 1SPK-M. The current-carrying system of a section of cable 1SPK-M, ∿8 m long, consists of two coaxial copper tubes coated with a superconductor. On one side, part of the current-carrying system is led into the zone of room temperature, and it forms the current lead-ins. A detailed description of the design is contained in [6].

The steady-state temperature distributions along the inner core were experimentally investigated at different pressures of the stagnant helium in the space of the inner core. The parameters of the cryoagent at the inlet to the cable were maintained constant $G = 10^{-3}$ kg/sec, $T_{g0} = 8.7$°K, P = 0.4 MPa. The obtained temperature distributions are shown in Fig. 2. To determine λ_{equ}, the system of equations (3) was used.

The method of determining λ_{equ} consists in comparing the experimental data with the results of calculating the temperature distribution with the aid of the system of equations (3) for the section of the cable core on which the temperature gradient is small and the properties of the materials undergo little change. The solution of system (3) for the section of core was obtained in the form of a sum of exponential curves; it is not presented here because it is too cumbersome. The equivalent differential thermal conductivity λ_{equ} was viewed as matching parameters and was determined from the condition of minimum of the error function $\Phi(\varepsilon)$ (least-squares method):

$$\Phi(\varepsilon) = \sum_{i=1}^{n} (T_{pi} - T_{ei})^2, \tag{9}$$

where $\varepsilon = T_{pi} - T_{ei}$; T_{pi} is the temperature calculated with the aid of system (3) for the given point i with the test value λ_{equ}; T_{ei}, experimentally obtained temperature at the point i; and n, number of experimental points.

The data were obtained in the form of a ratio of the differential equivalent thermal conductivity λ_{equ} to the thermal conductivity of helium λ with pressures of 0.1, 0.2, and 0.3 MPa, and they were $1.4 \cdot 10^5$, $2.5 \cdot 10^5$, and $3.1 \cdot 10^5$, respectively.

These data with an error of up to 30% are generalized by the expression

$$\frac{\lambda_{equ}}{\lambda} = 4.2\,\mathrm{Ra}^{0.63},\ 1.4 \cdot 10^7 < \mathrm{Ra} < 6 \cdot 10^7, \tag{10}$$

where $\mathrm{Ra} = g\beta d^4 \frac{dT}{dx} \Big/ a\nu$.

The dependence of the thermal conductivity of stagnant helium $\lambda_{equ}s_3$ on the longitudinal coordinate, calculated for curves 1, 2, and 3 in Fig. 2, is shown in Fig. 3. It can be seen from the figure that the thermal conductivity changes weakly along the core, and on the lead-in it drops linearly toward the "hot" end.

It can be seen from a comparison of the thermal conductivity $\lambda_1 s_1$, which for the core is $\sim$1 W·m/°K, with the values of $\lambda_{equ}s_3$, shown in Fig. 3, that the equivalent thermal conductivity of stagnant helium in the investigated pressure range is much higher than the thermal conductivity of the core. In the calculation, the thermodynamic parameters of helium were taken from [7]; the thermal conductivities of copper, having $\rho_{300°K}/\rho_{4.2°K}$, were taken from [8].

Experimental Verification of the Formula. To calculate the maximum temperature of the core of 1SPK-M with the aid of formula (8), it is necessary to determine the temperature at which the properties of the materials of the wall and of the cryoagent have to be taken. On the basis of considerations which will be presented below, the properties of the materials of the core have to be taken at the maximum core temperature, and for the lead-in at the temperature of the central cross section of the lead-in.

In fact, the part of the core that is far from the place of contact of the cores with the lead-ins has little effect on the maximum temperature; therefore, in calculating the maximum temperature of the core and the equivalent thermal conductivity contained in formula (8), the thermophysical parameters of the materials are taken at the maximum core temperature, and the temperature gradient is taken in the cross section corresponding to this temperature. The heat flux from the side of the lead-in to the core of the superconductor is affected by the thermal conductivity of the entire lead-in. In calculating $\lambda_{T,equ}$ we have to average either with respect to T or x. In this case it can be seen that these averaging methods differ only slightly from each other, and in the calculations we took $\lambda_{T,equ}$ in the central section of the lead-in.

In calculating the maximum core temperature by formula (8), the iterative process has to be used. First we must specify the temperatures and gradients in the section lead-in—core and in the central section of the lead-in; then we calculate by formula (10) the differential thermal conductivities of the stagnant helium for the core and the lead-in, and by formula (8) we calculate the maximum core temperature.

The new values of temperatures and gradients are determined in the following manner. With the aid of (7) we calculate the thermal flux in the section lead-in—core. The temperature gradient in the section lead-in—core is calculated by the formula

$$\left.\frac{dT_w}{dx}\right|_{x=l} = \frac{Q_{\text{T}}(l)}{\lambda s + \lambda_{\text{equ}} s_3}.$$

The temperature and the temperature gradient in the central section of the lead-in are calculated by formulas obtained from the solution of the system of equations (4) on condition of ideal heat exchange:

$$T_{\text{T}w}\left(\frac{l_{\text{T}}}{2}\right) = \frac{T_{\text{T}w}(l_{\text{T}})\left(\exp\frac{g}{2} - 1\right) + T_w(l)\left(\exp g - \exp\frac{g}{2}\right)}{\exp g - 1},$$

$$\left.\frac{dT_{\text{T}w}}{dx}\right|_{x=\frac{l_{\text{T}}}{2}} = \frac{[T_{\text{T}w}(l_{\text{T}}) - T_w(l)]\exp\frac{g}{2}}{l_{\text{T}}(\exp g - 1)}.$$

With the new values of temperatures and temperature gradients in the central section of the lead-in and in the section lead-in—core, we calculate the differential thermal conductivities of stagnant helium, the maximum temperature, etc. The calculation process is continued until two successively calculated values of maximum core temperature differ by a specified value.

The results of the calculation of the maximum core temperature by formula (8) and the experimental data differ by less than 14%.

Calculation of the Maximum Temperature of Core SPK-100. The lead-in of the SPK-100 is analogous in design to the lead-in of the 1SPK-M. The temperature of the "hot" end of the lead-in of the SPK-100 may change from 250 to 300°K. In the calculations the temperature of the "hot" end was taken as 250°K. The indeterminacy of 50°K in specifying the temperature of the "hot" end causes an error in determining the maximum core temperature of less than 16%; this follows from the direct substitution into the left-hand side of relation (8).

The calculations of the maximum core temperature by formula (8) for the SPK-100 were carried out for two cases: for a pressure of 0.1 MPa of the stagnant helium, and for a vacuum inside the inner tube of the coaxial. The following data were used: T_{w0} and $T_{\text{T}w}(l_{\text{T}})$ equal to 14 and 250°K, respectively; G and $G_{\text{T}} = 6.3\cdot10^{-3}$ and $0.08\cdot10^{-3}$ kg/sec, respectively; s, s_{T}, and $s_3 = 0.725\cdot10^{-3}$, $0.725\cdot10^{-3}$, and $4.3\cdot10^{-3}$ m^2, respectively; $l_{\text{T}} = 2.7$ m; $\alpha = 35$ W/(m$^2\cdot$°K); p = 0.25 m.

If in the inner cavity of the coaxial there is a vacuum, then the inner and outer cores have approximately the same conditions. It may therefore be assumed that the same amount of helium is required for cryostating each core. The maximum core temperature may therefore be calculated by formula (8), with half the flow rate of cryoagent considered to be supplied to the cable.

In the calculation of the maximum core temperature and of the heat flux in the section inner core—lead-in, at a pressure of 0.1 MPa inside the inner tube and with exhaustion of the inner cavity, the following results were obtained: $T_w(l) = 35.3$ and 17.6°K; $Q_{\text{T}}(l) = 185$ and 11.2 W, respectively. In the calculation three iterations were required.

NOTATION

T, temperature, °K; λ_{equ}, equivalent differential thermal conductivity, W/(m·°K); d, diameter, m; α, heat-transfer coefficient, W/(m$^2\cdot$°K); p, perimeter, m; c_p, heat capacity at constant pressure, J/(kg·°K); q, heat flux density, W/m^2; q_1, outer radial heat influx to cores, W/m; Q, axial heat flux in the section core—head-in, W; G, flow rate of cryoagent, kg/sec; s, cross-sectional area, m^2; k, coefficient of thermal resistance, m$^2\cdot$°K/W; P, pressure, MPa; α, thermal diffusivity, m^2/sec; β, thermal coefficient of volume expansion, 1/°K; λ, thermal conductivity, W/(m·°K); ν, kinematic viscosity, m^2/sec. Subscripts: w, wall; g, liquid; T, current lead-in; p, calculated value; e, experimental value; 1, outer core and cryoagent; 2, inner core and cryoagent; 3, stagnant helium; 0, value of the parameter at the cable inlet.

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EFFECT OF THE GEOMETRIC CHARACTERISTICS OF A MULTIJET MIXING CHAMBER ON HIGH-TEMPERATURE HEAT EXCHANGE IN A CHANNEL

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UDC 536.244/536.45

The work entailed the experimental determination of the effect of the ratio of the diameters of the nozzles of electric-arc gas heaters to the channel diameter of a multijet mixing chamber on the heat exchange of a high-temperature air stream with the channel walls.

Multijet mixing chambers [1] are now widely used in installations for the investigation of the heat exchange of high-temperature heat carriers with channel walls [2, 3], and they are also used as the basic units of technological plasma apparatuses [4, 5]. These mixing chambers are short (1-2 bore diameters) water-cooled axisymmetric channels with circularly uniformly spaced holes for the inlet of several plasma jets from electric-arc gas heaters (EAGH). As a result of the collision of the jets at the center of the chamber, a high-temperature gas stream forms that moves into the channel, which is a continuation of the chamber.

The geometric characteristics of multijet mixing chambers and the methods of processing the experimental data in different investigations differ from each other. There is practically no information on the effect of the conditions of formation of the plasma stream in a multijet mixing chamber on its heat exchange with the channel walls. On the other hand, even for less complex conditions of formation of a high-temperature gas stream at the inlet to the channel, it was established that a change of the geometric characteristics at the initial section of the channel, in particular, of the inlet angle [6], leads to a change of the intensity of heat exchange between the gas and the channel walls. In connection with that, the present work investigates the effect of the slope of the plasma jet to the axis of the mixing chamber φ and the ratio of the diameters of the EAGH nozzles to the diameter of the mixing-chamber channel d/D on the high-temperature heat exchange.

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